Numerical model for hollow electron beam collimator

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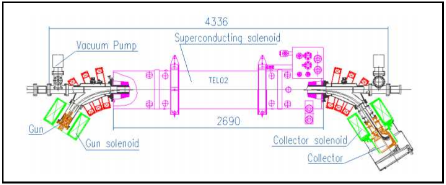
Introduction (1)

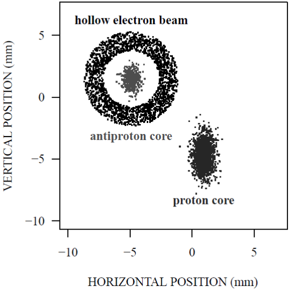


Advantages:

- no material damage
- low impedance
- controled by magnetic field
- transverse kicks are not random
- resonant exitation

Disadvantages:

- small kicks
- electron beam imperfections



Introduction (2)

Goals

- develop a numerical model for hollow electron collimator to be integrated into beam dynamics simulations for Tevatron
- compare with experements for Tevatron

Trajectories integration

First order symplectic drift-kick integrator,

$$\boxed{x'_{n+1} = x'_n + \Delta x'(x_n), \qquad x_{n+1} = x_n + x'_{n+1}\Delta s}$$

kicks are expressed via electron beam rest frame transverse electric field E_x and E_y,

$$\boxed{\Delta x' = \Delta s \frac{e\gamma_e(1 \pm \beta_e\beta_{\bar{p}})}{c\beta_{\bar{p}}p_{\bar{p}}}E_x, \qquad \Delta y' = \Delta s \frac{e\gamma_e(1 \pm \beta_e\beta_{\bar{p}})}{c\beta_{\bar{p}}p_{\bar{p}}}E_y}$$

- γ_e – electron relativistic factor
- β_e – electron relative velocity
- $c\beta_{\bar{p}}$ – antiproton beam velocity
- $p_{\bar{p}}$ – antiproton momentum

- $-$: $\vec{v}_{\bar{p}}\vec{v}_e > 0$
- $+$: $\vec{v}_{\bar{p}}\vec{v}_e < 0$

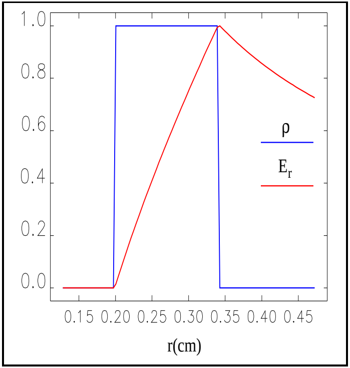
Kick from ideal charge distribution

$$\boxed{\Delta r' = \begin{cases} 0 & \text{if } r < r_1 \\ 2\Omega_e \frac{r^2 - r_1^2}{r(r_2^2 - r_1^2)} \Delta s & \text{if } r_1 < r < r_2 \\ 2\Omega_e \frac{1}{r} \Delta s & \text{if } r > r_2 \end{cases}}$$

$$\Omega_e = 0.3 \times 10^{-7} \frac{I_e \ [\mathrm{A}]}{p_{\bar{p}} \ [\mathrm{GeV/c}]} \gamma_e \frac{1 + \beta_e \beta_{\bar{p}}}{\beta_e \beta_{\bar{p}}}$$

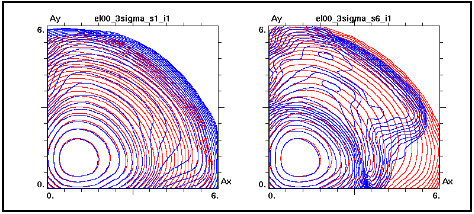
I_e – electron beam current, $p_{\bar{p}}$ – antiproton momentum

r_1 – inner beam radius, r_2 – outer beam radius

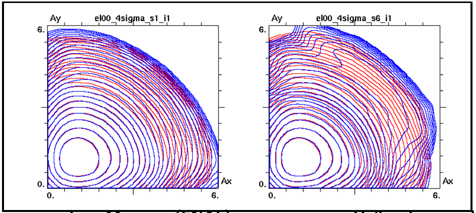


Simulation of ideal lens (1)

$r_1 = 3\sigma_y$



$r_1 = 4\sigma_y$



Antiproton beam size:

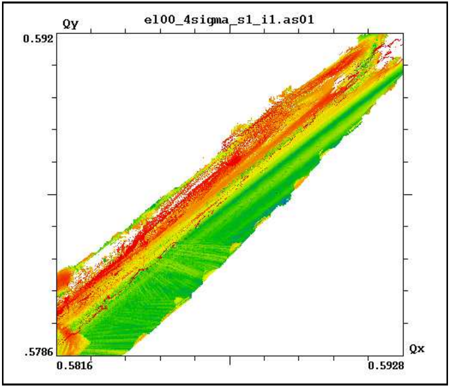
- $\sigma_x = 0.32$ [mm]
- $\sigma_y = 0.50$ [mm]

Simulation parameters :

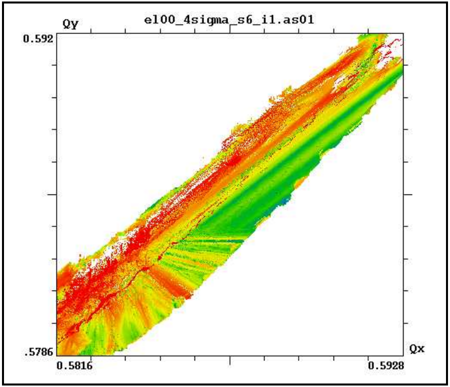
- $I_e = 1.0$ [A]
- $\beta_e = 0.2$ (about 10 [keV])
- $L_e = 200.0$ [cm]
- 3×10^6 turnes (about 1 minute)
- pulse pattern 1/1 (left) and 1/5 (right)
- lattice turns $Q_x = 0.578$, $Q_y = 0.575$

Simulation of ideal lens (2)

FMA: $r_1 = 4\sigma_y$



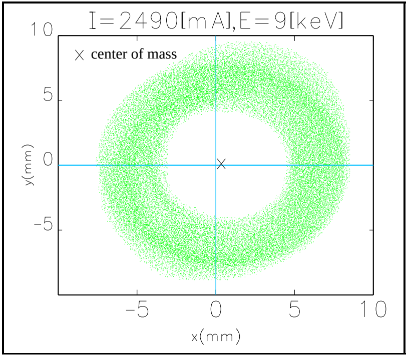
$skip = 1/1$



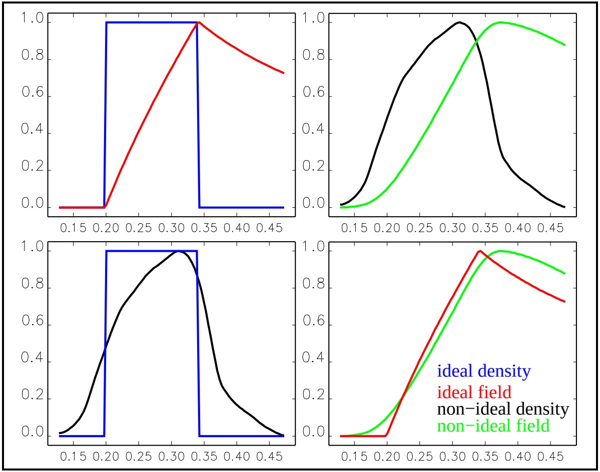
$skip = 1/5$

Sources of imperfections

- lack of the axial symmetry
- deviations in radial distribution
- electron beam bends
- longitudinal density variations
- beam misalignment



Radial model (1)

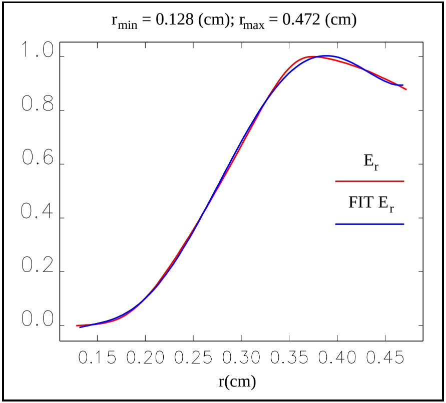


This profile corresponds to electron current $I_e = 2.0[A]$.
Typical current in experements is $I_e = 0.5[A]$.

Radial model (2)

$$\boxed{\Delta r' = 2\Omega_e \frac{f(r)}{r_m} \Delta s}$$

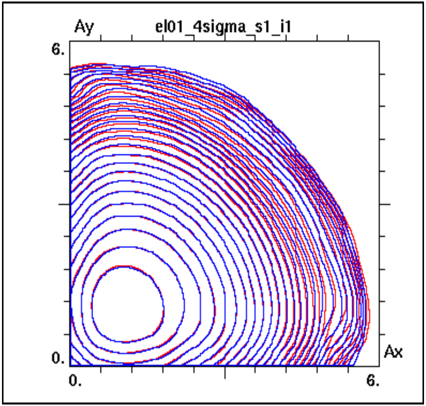
- $f(r)$ – interpolated polynom
- $f(r_m) = 1.0$



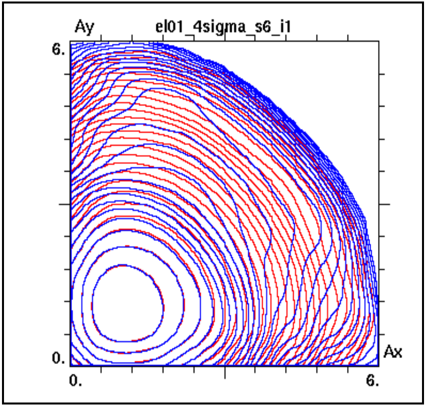
$$\boxed{f(r) = -2.6 + 63.0r - 584.0r^2 + 2512.9r^3 - 4838.9r^4 + 3405.0r^5}$$

Simulation of radial model

$skip = 1/1$



$skip = 1/5$



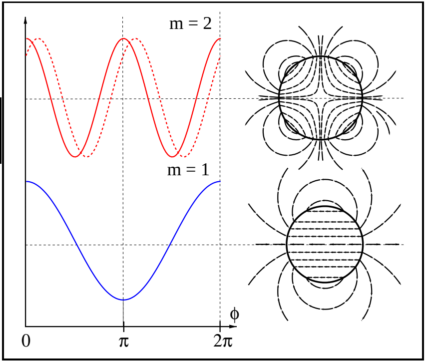
Cylindrical model (1)

$$\boxed{\rho(r,\theta) = \frac{f(\theta)}{2\pi r_b}\delta(r - r_b)}$$

$$\Delta x' = \Omega_e \Delta s \xi_m \begin{cases} -r^{m-1} r_b^{-m} \cos((m-1)\theta + \delta_m) & \text{if } r < r_b \\ r^{-m-1} r_b^{m} \cos((m-1)\theta + \delta_m) & \text{if } r > r_b \end{cases}$$

$$\Delta y' = \Omega_e \Delta s \xi_m \begin{cases} r^{m-1} r_b^{-m} \sin((m-1)\theta + \delta_m) & \text{if } r < r_b \\ r^{-m-1} r_b^{m} \sin((m-1)\theta + \delta_m) & \text{if } r > r_b \end{cases}$$

- r_b – cylinder radius
- m – harmonic number
- ξ_m – relative harmonic amplitude
- δ_m – harmonic phase



$m = 1$ – dipole, $m = 2$ – quadrupole

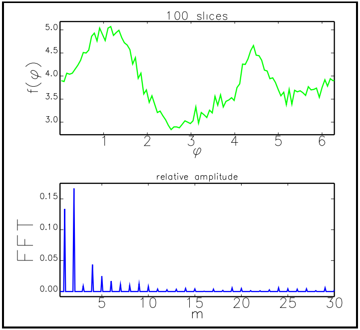
Cylindrical model (2)

Harmonics parameters for real beam profile

m	$\xi_r(r_b = 2.0mm)$	$\xi(r_b = 10.0mm)$	δ
1	0.179	0.90	-0.9
2	0.136	3.34	-2.35
3	0.037	4.73	3.10
4	0.026	16.10	1.99
5	0.008	24.48	-1.20

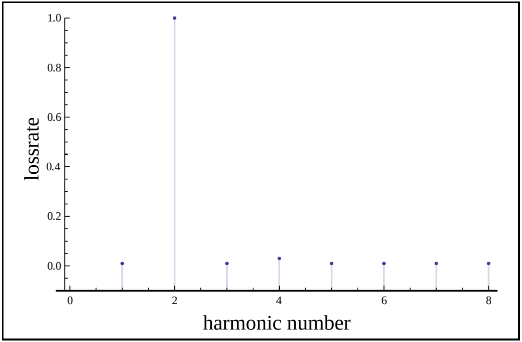
The dominant harmonic is the second one (quadrupole) about $15 - 20\%$.

Angular distribution function and it's FFT

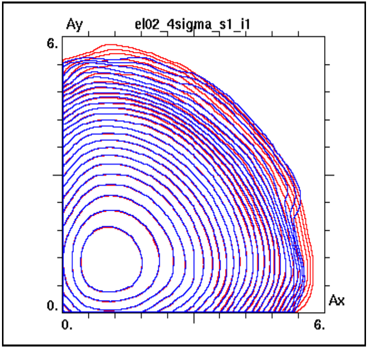


Simulation of cylinder model (1)

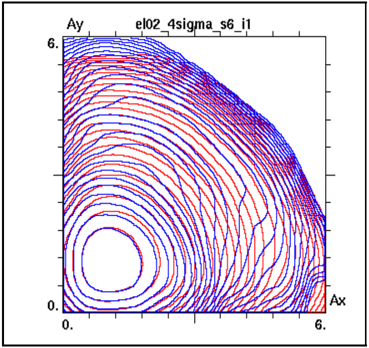
Simulations for different harmonics were performed ($m = 1 \ldots 20$) with pulse patterns $skip = 1/1, 1/2, \ldots, 1/8$.



Simulation of cylinder model (2)



skip $= 1/1$



skip $= 1/5$

Conclusions

- several transverse numerical models were developed
  - ideal element
  - element with radial imperfections
  - element with angular imperfections
- simulations were performed for Tevatron structure
  - ideal element behaves as it should
  - no significant emittance growth from imperfections
  - angular imperfections only significant for quadrupole harmonic with $skip = 1/5$

Plans

- comparison with Tevatron experimental data
- integration into SixTrack, MAD
- lens misalignment
- edge effects
- high order integrators
- 3D field from Warp code

Thank you for your attention!